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Scientific Models and Games of Make-Believe: A Modal-Logical Perspective

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Introduction

The fictionalist approaches to scientific models are legitimized by the abstract or idealized aspects of the properties which those models are about. Many concrete scientific illustrations among various fields support the argumentations consisting in comparing a model and a fiction. For example, some models deal with perfectly spherical planets with uniform mass density in physics, with constant population growth in biology, with the division of the atmosphere into cubes in climate models in meteorology, or with wireless sensor networks unaffected by radio interferences in computer science. Generally, those simplifications (abstractions and idealizations) aim at making the model useful in a plurality of possibly compatible situations, even if, paradoxically, such a process leads to a model which does not seem to be realizable. Just because a model has a limited form, making abstractions (by just taking into account a handful of properties and leaving aside all the others which are not relevant to that model), and conducting idealizations on the retained properties to have different concrete cases "represented" by an ideal one, the deeply

modal nature of such a model is explained, insofar as counterfactual situations have to be studied. Indeed, the parameters which have not been retained for the modeling will have a particular value in each situation compatible with that model. I will argue that an analysis of scientific models in terms of possible worlds, based on some important notions of modal logic, enables to understand the compatibility between a model and a possible world in an interesting way, thus criticizing a kind of "naive" fictionalism which consists in reading into the statements of the model-description too literally and according to which a model would focus on unique fictional entities.

Furthermore, among all of those fictionalist approaches, some resort to the notion of game of make-believe developed by Kendall Walton[1]. However, they do not accord on how to use it. For example, Roman Frigg and Adam Toon proposed an analysis of models in terms of games of make-believe as early as 2010 without formulating the same explanations for the relations between models and target-systems. In Walton's view, a game of make-believe consists in taking on a specific position when faced with a work of fiction: the point is not to ask what a model is about, it is to imagine the fictional truths prescribed by that model. Besides, Walton distinguishes between authorized and unauthorized games. That difference is important to understand what a "representation" is according to Walton (in this paper, every allusion to the notion of representation will refer to the definition set by Walton). Indeed, I will bring to light some specific games occurring during the manipulation of models, and thus I will uphold the epistemological (and non-ontological) nature of the actual exemplifications of fictional properties: there is an extrapolation when actual objects are considered to be exemplifications of that kind of properties.

From a scientific point of view, using the notion of authorized game seems preferable because the role of the individual agent is thus reduced in that kind of extrapolation. Indeed, in the words of Walton, a work of fiction is a prop in an authorized game only if the community concedes that the work has been created in order to be such a prop.

1 Walton, 1990.

Otherwise, if a work of fiction is used in a wrong way or in an unfair manner, the game is unauthorized (or unofficial). It means that an inner circle of persons can be the origin of such a game, without the agreement of the other members of the community (or even without them being informed). Therefore, I will explain that abusive extrapolations occur during the application of a model to an actual situation, so as to emphasize the purely epistemic construction of a set of actual objects supposed to be exemplifications of fictional properties; the latter being considered as world-lines within the meaning defined by Jaakko Hintikka.

Thus, I will be opposed to a number of aspects of Walton's interpretations proposed by Frigg and Toon.

According to Frigg, a model-description introduces a fictional entity named "model-system" representing the target system. I will argue that such an analysis of a description is not relevant because it consists in reading its statements literally, by considering that every linguistic expression is aimed at a kind of entity which would satisfy that expression strictly and simply (just as if the predicate "being red" could embody itself as an object which would be red only, without any shape or matter, or any other property). By assuming the unicity of such an object, that interpretation is incompatible with modal logic the tools of which I will use in order to take into consideration the multiplicity of possible worlds making true the propositions delivered by the model-description.

As for Toon, the relation between model-description and target-system is direct: from such a description, one has to imagine that objects in the actual world possess properties they do not have in reality. I will examine the manner with which Toon interprets Walton's notion of representation and I will recall the idea that a representation can only emerge in the framework of very specific games of make-believe as Walton describes them. Furthermore, I will put the use of a fiction-operator by Toon to the test of modal contexts.

Finally, I will suggest an interpretation of Walton that will be consistent with modal logic (Walton himself does use the notion of possible world). Standing out against a "naive" fictionalism the features of which I will detail, I will consider there is a direct link between model-descriptions and possible worlds in accordance with the semantic relation of

the satisfaction of a description by a class of worlds (a key concept in modal logic). In addition, in order to pursue a major aspect in Frigg's work concerning the similarity between properties, I will introduce an epistemological link between the worlds, by defining the properties so as to be able to talk about the same property from a world to another one, by means of the concept of properties as world-lines that I shall present; that concept enables to comprehend a property as a function whose value in a world is a subset of its domain, made up of the elements that instantiate the property in that world. Those notional lines have values in diverse worlds. For example, in the case of an idealized property, and by strictly respecting the principles established in Walton's work, that property has set values in fictional worlds, in the framework of authorized games. But I will argue that that property can have a non-empty set-value in the actual world only as part of an epistemological extrapolation of the related world-line, adhering to what I shall name the dispositional profile of that property described in the model-description.

1. "Naive" fictionalism

A certain fictionalist point of view about scientific models could be branded as "naive" because it consists in considering that what the models are about is a simple and unique fiction, which would satisfy the stipulated properties in the model only, and no other properties. The general terms of a linguistic description of a model (a model description), the miniature objects of a mock-up, or the colored diagrams of a data model are interpreted as designating fictional entities. For example, in the case of a model consisting of a text, that is to say of a set of statements, a general term is considered as an individual term. But according to this view, if that term has well and truly a unique designation, the pointed individual is particularly peculiar because it is an incomplete "naive" fictional entity which only satisfies a small number of properties. More precisely, that entity only possesses the properties shared by all the objects it is supposed to represent. For example, a model which is supposed to be about the Triangle in general defines such an abstract entity as "the" Triangle which would have no side lengths nor angles determined, without color or substance, without being right-angled or isosceles, and so

on, but which would only be abstractly defined as a plane figure with three sides, and therefore having no other property (I will clarify the idea of "incomplete" object below). That is why, according to this view, the general terms can be considered as fictional names, to comply with the terminology of Gregory Currie[2], just like the names of literary characters such as Sherlock Holmes.

Obviously, from a realist point of view, unlike with a fictional character, it seems that a theoretical term does not target just one entity, but some real objects. However, from a purely fictionalist perspective, even if scientific models aim at focusing on the world, they are literally about a fiction. Thus, the relation between a model and a target system is not direct: the fictionalist view is clearly contrary to the realist one. Using the remarks of Carsten Held[3]: "the intention is to describe real objects, but reference to them is mediated by an abstract entity", even if, as Held points out, that entity does not belong to the class of the objects it represents (just like "the" Triangle is not a member of the set of real triangular figures).

That kind of abstract entity, or more precisely of fictional object according to a fictionalist view in keeping with a Meinongian tradition, is considered as being "incomplete", that is to say not entirely determined. Edward Zalta or Terence Parsons[4] notably share the opinion that an objet which has been generated from a fiction is not complete, unlike real objects, because a small number of properties has been stipulated with regard to them. In other words, according to Parsons[5], there is always at least one property whose neither assertion nor negation can be ascribed to an incomplete object. If, within the framework of a description conducting an abstraction, it has not been stipulated that the fictional object possesses a given property *P*, then, according to that view, it is not relevant to declare that the object has or does not have the property *P*. Thus,

2 Currie, 1990, 128.

3 Held, 2009, 146. Held's conception consists in performing mediation via model entities: "Expressions exposing the model refer to the model entities, which in turn are meant to represent worldly phenomena" (2009, 151).

4 Yagisawa 2014.

5 Parsons, 1980, 231.

as Zalta[6] sums up, strictly speaking, such an object only possesses the properties it was given within the framework of the fiction. For example, Sherlock Holmes has no more properties than those explicitly described in some statements in Arthur Conan Doyle's writings. Thus, as Richard Mark Sainsbury[7] stresses: "It's not the case that, according to the Holmes stories, Holmes had an odd number of hairs on his head when he first met Watson" (neither that he had an even number of them).

The point of that kind of fictionalism is, on the one hand, to take into account the issue caused by the simplifications made within the framework of modeling and, on the other, to show the representational power of the model-descriptions thus considered. Indeed, the created fictional entity rigorously satisfies idealized properties thanks to its fictional nature, and makes it possible to represent a large number of real objects thanks to its incompleteness.

The properties that the scientist leaves aside by abstraction during modeling are precisely those properties about which it is impossible to tell whether the generated fictional object possesses them or not. On the contrary, considering one of those properties, any real object possesses either that property or its negation.

For example in physics, if a model defines its object with two properties, such as "being spherical" and "having uniform mass density", then that fiction (incomplete by nature) which the laws of models are literally aimed at, is a mediation, a prism between that model and, for example, any actual ball which satisfies the two definitional properties, as well as any other property (for example, the ball may be any color since that type of property is not specified in the model-description). The way an actual ball can satisfy such properties will be analyzed below.

Furthermore, such a fictionalism can also be applied to models dealing with relational properties. At this stage, what is created is a fiction about several objects between which some relations lie. The same argument according to which a fictional object is incomplete (because generated by a finite number of properties) can be used regarding the pos-

6 Zalta, 1988, 126.

7 Sainsbury, 2010, 77.

sible fictional world described by a finite number of statements: more than an object satisfying a description of an entity, it is now a fictional world which satisfies a finite number of propositions. Thus, such a world understood as a medium between a model and the actual world should not be considered as a world seen as a whole universe about which any statement is either true or false: it should be called "small world" since it is incomplete (that is to say not entirely determined by the model-description). That expression, borrowed from Jaakko Hintikka[8], designates a world which makes only a limited number of statements true[9]. In other words, a small world does not verify the law of excluded middle; for any proposition p, it is not necessarily the case that in a small world w, the disjunction $p \vee \neg p$ is true in w (that is to say $w \not\models p \vee \neg p$).

A fictional object satisfies exactly the laws of the model in the framework of which it has been created, but one can question the usefulness and relevance of such a fiction; indeed, the scientists aim at studying the actual target systems or possible ones which are entirely determined. Unlike in a "naive" fictionalism, one can consider that a general term in science is "general", not because it designates an incomplete entity, but because it designates a set of complete objects. It could thus be argued that an "object", as Kendall Walton[10] named it, is an entity created through a kind of strict reification of a description limited to a finite number of propositions (like in the case of "the" Triangle). But Walton[11] himself rejected the existence of such an object. Indeed, one can doubt the epistemological relevance of an incomplete fictional entity and question it altogether.

8 Hintikka, 2007, 62.

9 I borrow the term "small world" from Hintikka, but he used it to deal with situations limited on a spatio-temporal plane. A small world, he said, is bounded but not incomplete. The way I use the term must thus be regarded as different from Hintikka's; these are two different aspects, even if an isolated situation may also be incomplete.

10 Walton, 1990, 106.

11 As regards Walton's point of view about small worlds, it seems that fictional worlds already exist: "Fictional worlds, like reality, are "out there," to be investigated and explored if we choose and to the extent that we are able." (Walton, 1990, 42). Yet, later on he wrote: "Fictional worlds are sometimes impossible and usually incomplete, whereas possible worlds (as normally construed) are necessarily both possible and complete" (Walton, 1990, 64). I will compare below what Walton calls "work worlds" and small worlds.

First of all, if the notion of incomplete fiction justifies itself by the necessarily finite number of properties considered in a description, will it still be relevant when faced with properties resulting from those definitional properties? For the sake of clarity, the case of coextensive properties should now be considered. If a fiction describes its object as a creature endowed with a heart in our actual world, the generated object should satisfy the property "having a kidney", even if the latter is not part of the definitional properties. In other words, if one knows that "p implies q" and that p is a proposition present in the description that an object satisfies, it would be logically true that that object satisfies the proposition q. Thus, as I will explain below, a mere description does not generate a purely abstract fiction (an incomplete character or a small world) which satisfies only the properties explicitly given within the fiction.

Moreover, a model which describes "its" object as a perfectly spherical ball actually aims at encompassing a whole set of possible balls. The fiction covered by a model (in the sense of "naive" fictionalism) is a parasitic fiction insofar as it does not enable to provide a relevant view of (complete) possible worlds; yet, those are the worlds that the model is supposed to be about. It is an incomplete fiction which does not reflect scientific practice. It may be a remnant of a kind of platonism according to which an entity such as "the Ball" exists and has to be the thing designated by a scientific term. But what Walton proposed was a certain posture to be adopted towards a work of fiction rather than an answer to the issue concerning the denotation of general terms.

2. Walton: Games of make-believe

In the framework of an analogy between children's games and artistic activities Walton developed the notion of make-believe in order to unify the attitudes to be adopted towards works of fiction. Whether it be children playing "cops and robbers", or adults attending a play, or a girl tending to a doll, they all are supposed to get their imaginations going. The games of make-believe are a specific kind of imagination activity involving some "props". An example suggested by Walton is the game played by a group of children consisting in doing "as if" the tree stumps

were bears: "Let's say that all stumps are bears"[12]. The children do not just imagine bears: they imagine that the stumps are bears. Joining a game of make-believe means complying with the principles of generation (or "rules" of the game) by imagining what is prescribed. For example, a novel, understood as a prop, urges the reader to imagine what is described, to generate possible worlds compatible with the fictional truths prescribed within it.

However, according to Walton's distinction[13] about imagination, if there is a finite number of propositions in the work of fiction: firstly, one imagines those propositions in a deliberate way (in rigorously respecting the definitional properties), then in a spontaneous way. In other words, based on a work of fiction, a small world is deliberately generated, but a wider possible world is spontaneously generated (by determining some non-definitional properties).

Indeed, some fictional truths, different from those prescribed by the propositions in the description, are imagined in a spontaneous way like, at least, in the case of coextensive properties, as I pointed out above. The proposition according to which the imagined bear possesses a heart is fictional, but the fact that the gamers imagine it is not essential for the smooth running of the game; only the compliance with the explicit rules of generation matters (that is to say with the definitional properties in science, as I will remark below). Nevertheless, imagining "secondary" propositions can be useful within the game: the bear has a heart which has to be aimed at in order to kill the bear[14].

If that example of children's games regarding stumps enables Walton to illustrate a game of make-believe, it is important to understand that it is not a relevant example for what Walton means when using the term "representation". Indeed, he distinguishes two kinds of games: authorized and unauthorized ones (the latter are sometimes branded "unof-

12 Walton, 1990, 24.

13 Walton, 1990, 14.

14 Similarly, in science, spontaneous secondary fictional truths can be used to develop a model in which, for example, a planet is perfectly spherical with uniform mass density. The property of being a point mass can then be considered as a secondary truth about this planet.

ficial" by Walton in order not to imply something illicit)[15]. According to Walton, the notion of representation can only be broached in the framework of an authorized game and Walton specifies:

> "The stumps and cloud formations especially are likely to seem oddly sorted with representational works of art. I propose to understand 'Representation' in a way that will exclude them. The stumps are ad hoc props, pressed into service for a single game of make-believe on a single occasion. Dolls and toy trucks, by contrast, are designed to be props; they were made specifically for that purpose. That is their function, what they are for, as it is the function of chairs to be sat in and of bicycles to be ridden (…). I will call games of the kind a given prop has the function of serving in authorized ones for it"[16].

In other words, whether they be objects, novels or pictures, only the things created in order to prescribe fictional properties or have acquired that status within a community (by tradition or convention) are considered as props in an authorized game. Furthermore, a work of fiction has a representational function only within an authorized game, with a prop whose function is to prescribe certain fictional truths. That function can vary from a community to another. But if a prop has that prescribing function in an authorized game, then the stipulation of the rules is no longer necessary; as Walton pointed out: "This is like having an established language available to use for any conversation, rather than having to set up an ad hoc code for each one"[17]. Briefly, according to Walton, representations are 17 those things about which it is known that the function is to serve as a prop in games of make-believe: "Representations, I have said, are things possessing the social function of serving as props in games of make-believe"[18].

15 Walton, 1990, 406.

16 Walton, 1990, 51.

17 Walton, 1990, 53.

18 Walton, 1990, 69.

Yet, unauthorized games should not be dismissed because they do occur even beyond the field of art. Walton illustrated that clearly: "It is not the function of La Grande Jatte [by Georges Seurat] to be a prop in games in which fictionally hippos are wallowing in a mud hole, no matter what games people actually play with it. The hippopotamus game is inappropriate for the painting, unauthorized (in the sense defined earlier); to play it is to misuse the work"[19].

In terms of modal logic, one could say that the proposition according to which "a pair of hippopotamuses are wallowing" is true in a fictional world, but that world is not compatible with the fiction; the game which rests upon that painting as a prop and which consists in imagining a pair of hippos is an unauthorized game.

More generally, props are ad hoc (as in the case of the stumps), they are not representations because they are not used "appropriately"; it is not their function to serve as props in such games. Walton expressed that distinction between authorized and unauthorized games in terms of possible worlds. Indeed, he distinguished two classes of worlds: the work worlds and the game worlds. On the one hand, game worlds are generated by a game of make-believe; if that game complies with the function of the prop, the game is authorized. Otherwise (like in the case of the hippos), it is unauthorized[20]. On the other hand, work worlds make true the propositions prescribed by the prop only (regardless of any agent). That distinction matters because some truths in game worlds do not occur in work worlds. For example, the fact that an audience is afraid of a monster described in a work of fiction is not a truth in the worlds compatible with the fiction (the work worlds). Thus, a model-description is satisfied by a set of work worlds, while a model-description is the prop of a game of make-believe played by an agent, involving the creation of game worlds.

Among those game worlds, some are authorized, the others are not. The authorized games are those which respect the primary function

19 Walton, 1990, 60.

20 A game world is authorized if it has been generated by an authorized game. Similarly, a game world is unauthorized if it has been generated by an unauthorized game based on a prop whose function is not to serve as a prop.

of the prop (a model-description in the case I am interested in): if that prop has been created in order to prescribe some fictional truths, an authorized game consists in respecting and obeying those prescriptions, whereas an unauthorized one does not respect those prescriptions (and the agents themselves can imagine new prescriptions, even if the latter are in conflict with those of the prop, like in the case of the hippos).

Thus, what is fictional in work worlds is fictional in worlds of authorized games too. Indeed, one can understand the work worlds in the sense of Walton as small worlds which make true only the prescriptions of the work of fiction ("The work world includes only fictional truths generated by the work alone"[21]), while those prescriptions plus other propositions compatible with that work are respected in the game worlds of an agent. I will explain below why I reject that notion of small worlds; the worlds imagined by an agent are conceptually closer to what is called "possible worlds" in modal logic (notably regarding the deduction of propositions). Thus, game worlds are expansions of work worlds (those worlds which are strictly compatible with the fiction and are incomplete because they make neither true nor false propositions other than those stipulated in the description). The game worlds are relative to the more or less fertile imagination of the agents. As long as that expansion respects the model-description by not generating incompatible propositions, the game is authorized. But it becomes unauthorized if the agent takes the liberty to extrapolate, for example by imagining that the actual world satisfies the description while it does not satisfy the definitional propositions (notably in the case of an idealization). The only thing that the agent can do in an authorized way is imagine that a fictional world satisfies the model-description (but it is not the case of the actual world).

To sum up, Walton defines a representation only in terms of worlds generated by an authorized game based on a prop used in such a way that it is compatible with its function. For example, a fictional world generated from a tree stump is devoid of fictional truths; in that case, there is no world generated by an authorized game which would respect the non-representational nature of that prop. In most cases, it is possible for

21 Walton, 1990, 216.

two classes of worlds to be generated starting from the same prop: the work worlds and the authorized game worlds compatible with the functions of the prop, and the game worlds deriving from an unauthorized use of that prop. For example, toys are representations of other things, like a real truck or a genuine infant. So there are worlds generated by authorized games that are compatible with the recognized functions of the prop (doing as if the doll is crying), but from the same prop there are worlds generated by unauthorized games too (doing as if the doll is breathing fire). According to Walton, there is a representation when a prop (like a doll) is well known for its function within a community and when, from that prop, the agent imagines prescriptions compatible with its function. But imagining that an object possesses certain properties that it does not have in reality, that is to say following a game that the community does not recognize, is an ad hoc and unauthorized use, independently of any consensus or agreement from the other agents, especially when the rules of generation are not stipulated.

3. Scientific models and games of make-believe

Walton himself suspects that the notion of make-believe plays a role in the postulation of theoretical entities in science[22]. In the manner of Roman Frigg or Adam Toon, the conceptual combination between models and make-believe has been studied, even if Walton actually seemed opposed to that comparison:

> "It is not the function of biographies, textbooks, and newspaper articles, as such, to serve as props in games of make-believe. They are used to claim truth for certain propositions rather than to make propositions fictional. Instead of establishing fictional worlds, they purport to describe the real world"[23].

If there is an agreement about the point that scientific models aim at describing, explaining or even predicting the actual world, then it seems

22 Walton, 1990, 7.

23 Walton, 1990, 70.

inappropriate to use them as props for prescriptions of fictional truths which are only true in fictional worlds. Yet, as I have remarked above, lots of models are constructed on the basis of major simplifications. Even if the latter should not lead to supposing the existence of certain incomplete fictional entities, but aim at focusing on the largest possible number of situations, it seems that those simplifications lead to imagining certain things about those situations; or rather about the fictional situations possibly comparable to the target-situations.

Indeed, Walton made the difference between truth in fictional worlds and truth in the actual one, by distinguishing a statement such as "A bear was hiding in the thicket" and another such as "It is true in the game of make-believe that a bear was hiding in the thicket". Nevertheless, Walton refused to define a work as being a fiction merely from the intention of the author of that work, even if the use of a fiction-operator such as "Once upon a time" can indicate that intent. The status of fiction is related to a social community (more than to the wish of the author). For example, "the ancient Greek myths may have been nonfiction for the Greeks but fiction for us"[24]. Then, the issue is to know if a scientific model has to (or does not have to) serve as a prop in games of make-believe. The fictionality of a work does not fall under its factual truth, but is related to the attitude one adopts towards that work. When a model is about idealized properties for example, one has to imagine that kind of properties. Such a model is composed of prescriptions to imagine some fictional truths (true in fictional worlds). That is why some philosophers of science have extended that analogy between the use of a model and a game of make-believe.

3.1 Frigg's interpretation of Walton

Roman Frigg, notably, used Walton's theory about fictions in order to analyze scientific models as props in games of make-believe. Frigg emphasized that most model-descriptions begin with verbs such as "Consider" or "Suppose" indicating that it is an invitation to imagine things. Indeed, the use of those verbs could be compared to the use of a fiction-operator: the statements which are in its scope have to be imagined.

24 Walton, 1990, 91.

In such cases, the de *dicto* use of those "operators" is the sign for prescriptions in authorized games. "For this reason there is nothing mysterious about ascribing concrete properties to nonexistent entities"[25]. The agent, by respecting the primary meaning of those operators, imagines the prescriptions by following the rules of an authorized game. Yet, Frigg warned about the common meaning of the term "imagination"[26]. In the framework of a (authorized) game of make-believe, the imaginations are guided by the prop itself and the rules of generation. In other words, the imagination is governed by intersubjective rules which make sure that every agent imagines the same things, so that the model-descriptions are the same for everyone. That is certainly why Frigg seemed to suppose that a unique model-system is generated from a model-description. He wrote: "many different descriptions are meant to describe the same model system"[27], but he did not clarify if the opposite is true, namely: can several model-systems be described by the same model-description? In another paper, Frigg gave a clearer answer by using an indefinite singular article:

> "Like in literature, we introduce a model-system by giving a description: sentences specifying its features. Yet it is important to notice that the model-system is not the same as its description; in fact, we can re-describe the same system in many different ways, possibly also using different languages. I refer to descriptions of this kind as model-descriptions and the relation they bear to the model-system as p-presentation"[28].

Before studying what Frigg means by "p-representation", it is important to remark that, even if the imagination is supposed to be marked out by the model to the point that everyone imagines exactly the same thing, the issue related to the satisfaction of a description remains: generally in modal logic a model-description is not satisfied by a unique world,

25 Frigg, 2010a, 261.
26 Frigg, 2010a, 264.
27 Frigg, 2010a, 256.
28 Frigg, 2010b, 258.

but by a multitude of possible worlds. As for the nature of those worlds, the supposed uniqueness of a model system seems to recall the naive reading of a description (in the Meinongian sense): here, what is generated is not a unique object but a unique model-system. According to the modal approach presented below, I will consider that there is a whole class of model-systems starting from a model-description (according to Frigg's terminology). However, such a model-system seems similar to Walton's notion of work world, given that in both cases the model-description generates only one model-system or work world respectively. Yet, from another point of view about the nature of a model-system, a description generates a model-system which is unique but different from a small world; there is a wider world which goes beyond what was explicitly prescribed (following a spontaneous act of imagination in the sense I defined above): "Finding out what is true in a model system beyond what is explicitly specified in the relevant description is a crucial aspect of our engagement with the system"[29]. Indeed, as I have already written, the simplifications made within a model do not aim at generating a single possible world which would satisfy the mentioned properties only, and none other: those simplifications are supposed to make sure that the model will focus on the largest number of possible worlds. In any case, according to Frigg, all the statements in the model-description are about that kind of (unique but complete) model-system, just as a novel generates a character of fiction. That idea is related to Walton's works to the extent that, according to him, even if a real boy satisfied all the emitted statements regarding Tom Sawyer, the fiction "The Adventures of Tom Sawyer" should still be about that "object" in Walton's sense, that is to say about that character and not about that real boy.

In order to follow up on the works of Roman Frigg[30], it is important to study the distinction he made between p-presentation and t-representation. According to him, a model-description p-represents a model-system ("p" to emphasize that it is the prop, the model-description, which generates the game). If I insist on the point that a model description would p-represent a whole class of model-systems (and not just one of them),

29 Frigg, 2010a, 258.

30 Frigg, 2010b, 258.

I agree on using the notion of representation here, but it should be made clear that the relation of p-representation is the only authorized one in Walton's sense (unlike the t-representation suggested by Frigg that I will present below). However, one of the possible motivations which leads Frigg into considering the notion of model-system, and its uniqueness in particular, may consist in distinguishing between the substance and the form, thus adopting a semantic (and not a syntactic) approach to models. In my view, that unique model-system for each description seems similar to the entity created by a naive reading of a model-description. Even from a semantic view, two descriptions (in different languages for example) generate the same class of compatible possible worlds. The only authorized game consists in imagining that a model-description is satisfied in compatible fictional worlds. The accessible content of a model-description is not a model-system as Frigg suggested, but rather that class of compatible worlds. Yet, as I will argue, the relation between fictional worlds and the actual world is not of a representational nature: it is an epistemological extrapolation that I will illustrate by means of the properties as world-lines.

Frigg considered another relation of representation between a model-system and target-systems (possibly in the plural): the t-representation ("t" to emphasize that the representation now concerns the target-system). Yet, that relation should not be qualified as "representation" in Walton's sense, because it consists in an unauthorized game.

Indeed, I suggest that there is an important analogy between Frigg's distinction between p-representation and t-representation, and Walton's between representation and matching, as I will explain below. It is conceivable that, in the contemporary scientific community, models should be considered as works starting from which one has to imagine certain things (such as a body sliding on a surface without any friction), by the mere presence of fiction-operators ("Suppose that…"). From this perspective, it is actually coherent to consider a model-description as a prop in authorized games of make-believe and, as I argue, that several (not one) possible worlds make true the statements in this description. But these possible worlds compatible with fiction are fictional worlds, at least as regards the models with simplifications.

Before continuing this analogy, to better understand according to what aspects a scientific model can be considered as a support in a game of make-believe, I would like to examine further the use of a fiction-operator. A work of fiction contains fictional truths in the de *dicto* modality, such as "according to the fiction, there are unicorns which...", but it may be tempting, according to Walton, to think that these authorized representations also generate *de re* fictional truths, such as "there are unicorns, according to the fiction ..." (or like the tree stumps that are bears in the fiction). Clearly, let *F* be a fiction-operator (and the predicates H and C for "being a horse" and "having a horn" respectively), its *de dicto* use means: $F(\exists x\,(Hx \wedge Cx))$, while its de re use is symbolized as follows: $\exists x\, F(Hx \wedge Cx)$). In the former case, the objects compatible with the fiction only exist in the framework of that fiction. In the second case, there exist objects compatible with the fiction. In parallel, Walton distinguished representation from matching[31]. A representation results from a *de dicto* modality, consisting in imagining objects in the framework of an authorized game. A matching occurs in the context of a de re use, when, in an unauthorized game, real objects satisfy fictional truths, still within the framework of the fiction. In order to have a matching relation, the real object must possess, in a fictional way, all the properties prescribed in the work of fiction. In other words, according to the de re modality, there are real objects which satisfy the formula $F(Hx \wedge Cx)$. This does not mean that those objects are horned horses, but that they are according to the fiction, that is to say that some "counterparts"[32] of those objects have those properties in the worlds compatible with the fiction. "According to counterpart theory, the de re modal claim "*a* is possibly *F*" is true in world w just in case there is in some world a counterpart of a that is *F*"[33]. I will come back to that important point when I analyze what Toon suggested.

Thus, to continue the latter analogy, with the t-representation (standing between model-system and target system) and the notion of matching, it becomes obvious that one is studying a case of a non-represen-

31 Walton, 1990, 108.

32 I will return to this modal notion of the identity of an object across possible worlds by analyzing it in terms of individuals in Hintikka's sense.

33 Currie, 1990, 139.

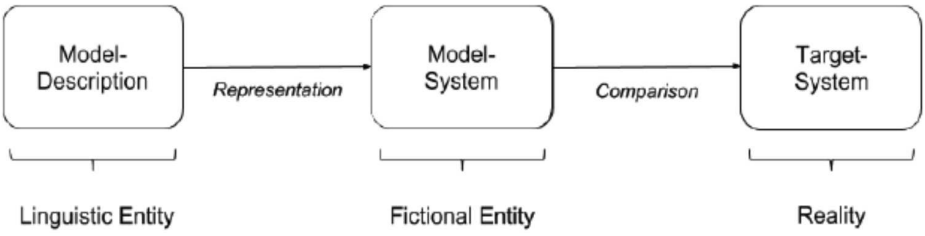

Figure 1: Frigg's conception

tational relation, if the model-description includes simplifications that real objects cannot satisfy. Indeed, it is actual objects that are now at the center of the game. The agent is requested not to generate possible worlds strictly compatible with the fiction, but to imagine that the actual world is also compatible with that fiction containing simplifications. In other words, in an authorized game, objects are imagined by assigning them properties; one supposes the existence of such objects only in the scope of the fiction-operator. By contrast, an unauthorized game consists in assigning fictional properties (idealized ones for example) to an existent object, outside the framework of the fiction. In my view, the nature of the relation between fictional worlds satisfying the description and the actual world is not representational (unlike what Frigg suggested by branding that relation as "t-representation"): this is an epistemological extrapolation that I will illustrate below with properties as world-lines. That is why I deliberately change (in Figure 1) the denomination of that relation between model-system and target-system ("comparison" instead of "t-representation") in order to remain consistent with Walton's work.

3.2 Toon's interpretation of Walton

In a way, Adam Toon is more radical and direct than Roman Frigg. He recognizes, as I have just done, that comparing a model and a target-system is an unofficial game in Walton's sense, but still thinks that the model-description is about the actual world and he even considers representations in that kind of cases when it would not be allowed by Walton. Furthermore, it is difficult to understand how Toon can guarantee that a given term in a model description designates an actual object, as if maybe one was guided by the designation of current terms in science; thus a model requiring an agent to imagine a perfectly spherical ball

would concern real balls directly. Yet, as I will remark, the recognition of a real object as an exemplification of a fictional property consists in a method bearing on the modal (or dispositional) profiles of those objects (and not simply on the analogy resulting from the use of the same term). How is one supposed to know whether the object one has to imagine from the model-description does exist in the actual world? Firstly, it is necessary to recognize the definitional (sometimes idealized) properties in an actual object. That recognition is not for granted, it has to be constructed. Moreover, still in a linguistic view, properties are independent of predicates, even if the latter are a means of access to properties: another means is the measure of causal powers which those properties bestow on their exemplifications, as I shall explain below. Finally, in an authorized game, one knows what is fictional: it is explicitly determined. But this is not the case in unauthorized games: a group of persons could be imagining that a tree stump is a bear without an external agent knowing the details of the game.

According to Frigg's point of view, there are two modes of games: one consisting in imagining a model-system which is authorized (p-representation) and the other consisting in establishing a relation between a model-system and the actual world which is unauthorized (t-representation). As for Toon, he eliminates the former kind of game because he considers the model-system as a useless fiction[34]. According to him, there is only one (unofficial[35]) game which directly links the model-description to the target-systems about which one has to imagine certain properties:

> "They [the descriptions] prescribe us to imagine propositions about the actual bouncing spring: we imagine of the actual bob that it is a point mass and of the actual spring that it is massless, and so on"[36].

34 Toon used the notion of possible world but thought, like Frigg, that a model whose propositions are satisfied by only one world. For instance, he wrote: "This is fictional in the world of our model" 2010b, 310.

35 "(...) the explanation we must offer is a little more problematic. One reason for this is that the pretence must now be understood to occur within an unofficial, rather than authorised, game of make-believe". Toon, 2012, 51.

36 Toon, 2010a, 84.

But, even if, otherwise, I consider as parasitic the idea of a unique small world making true a model-description, it seems essential to me, from a point of view that is compatible with modal logic, to be able to talk about the worlds within which the laws and properties prescribed by a model could be strictly satisfied, in order to compare at least one class of those worlds with the actual world. Indeed, in the manner of Toon, I suggest that a fiction consisting in a small world is parasitic from an epistemological point of view. But I insist on the fact that the fictional possible worlds which satisfy a description are complete worlds, conceptually closer to the actual world. For example, it is conceivable to compare those worlds from a structural point of view or by gradually bringing them closer to the actual world by eliminating one simplification at a time (like by setting a parameter at a time), or even to study the causal implications of a property in certain determined conditions (like in ideal cases).

Toon's statements concern the unauthorized games between model-descriptions and target-systems directly, whereas I argue that the generation of fictional worlds based on a model-description (or on a prop created to play such a role) is the only game that enables a representation to emerge. That reasoning rests upon Walton's works, especially those about the distinctions he made between representation and matching, and between the modalities of the use of the fiction-operator. However, Toon considers that a game of make-believe consisting in imagining certain things about an actual object can lead to a representation. The starting point is the same: the model-description is the prop in a game of make-believe. Frigg can talk about representation legitimately (a fictional entity is generated and one ascribes fictional properties to it). But, in suggesting that the prescriptions delivered by that description are about the actual world (even in cases of simplifications), Toon should not be "authorized" to brand that direct relation as "representation" in Walton's sense[37]. Toon wrote:

> "Usually, Walton thinks, when we read a linguistic work of fiction that uses proper names, we take ourselves to be pre-

37 Toon, 2010b, 305: "I believe we may regard scientific models as representations, in Walton's sense".

> scribed to imagine things of the normal referents of those names. On this view, the above passage represents (the actual) St. Paul's, because it requires readers to imagine certain things of St. Paul's"[38].

However, this does not seem to meet the ideas of Walton:

> "Authors sometimes model characters on people with whom they are familiar, or fictional events on actual ones. But this does not make the models objects of the authors' works; no fictional truths about them need be generated (…). David Copperfield is in one sense "autobiographical." But it need not be regarded as generating fictional truths about Charles Dickens"[39].

Thus, Walton distinguishes the truth in a fictional world generated from a description, from the truth in the actual world. A description prescribes certain things to be imagined and this generates a fiction. Even if that description contains common nouns (such as "ball") or proper nouns (such as "St. Paul's Cathedral"), the properties prescribed are about those fictions, and not about actual objects or persons that some think they recognize (this is an unauthorized game which does not enable to talk about "representation", as seen above). For example, an inappropriate use of a work of fiction could consist in ascribing a character trait (a property) to an actor playing a role rather than ascribing it to the fictional character which the work of fiction is about. Again, I suggest that such a game consisting in ascribing fictional properties to real objects is not official and that no representation in Walton's sense can emerge in such a framework. However, I consider that ascription as an unauthorized extrapolation which emphasizes the major epistemological work which consists in recognizing a fictional property (described in a model) in an actual target-system.

Moreover, Toon admits that the game consisting in ascribing fictional properties to an actual object can only be done through an imaginative

38 Toon, 2010b, 306.

39 Walton, 1990, 112.

process, since a real ball is not perfectly spherical, for example. Or, to return to the quote from Toon, this is not about the real monument but about those generated from the literary description; St. Paul's Cathedral is not actually destroyed, but there are possible worlds in which the fictional objects generated from the description are. The point consisting in naming those fictional objects as "counterparts" of the real cathedral is not granted and has an epistemological cost, as I will explain when I propose a possible use of the de re modality in terms of individuals. Faced with that difficulty, Toon could respond, for example, that one just pretends to assert that the cathedral is destroyed. So then this consists in using a fiction-operator as follows: "According to Wells' story, St. Paul's Cathedral is destroyed". However, such a statement does not generate a small world, but a multitude of complete possible worlds, each of them making true that proposition (in different manners for example). The authorized game generates imaginary versions of the cathedral about which one can imagine certain truths (even fictional ones). But as soon as a statement is taken out of its context, that is to say taken out of the scope of the fiction-operator, that statement becomes false (the actual cathedral is not destroyed). Besides, as Walton contends and as I have alluded to above, even if a real object satisfied a model-description, the fictional truths would still concern the imagined object, and not the real object:

> "Suppose that Tom Sawyer, the character in The Adventures of Tom Sawyer, has a double in the real world. There happens actually to have been a boy of that name who was and did everything Mark Twain's novel has the fictional Tom Sawyer being and doing – a boy, in other words, whom the novel matches (…). The Adventures of Tom Sawyer is not about this actual boy. He is not one of its objects (…) Mark Twain's novel does not prescribe any imaginings about the real-world counterpart of his character"[40].

40 Walton, 1990, 109.

Once one prescribes properties to an object while it does not possess them, one is not in the category of scientific writings or reports of facts any longer: one creates a fictional entity which possesses those properties, unlike the object (or the class of objects) which was maybe the starting point for the creation of that entity.

Toon's point of view can be expressed even more clearly by using Walton's terminology. First of all, the actual world is not a member of the work worlds: for example, the actual world does not meet certain idealized propositions from the model-description and, particularly, it is a complete world and makes true other propositions than those exclusively prescribed. But could the actual world be a member of the game worlds? Concerning the game worlds generated in an authorized way, the answer is no since an authorized game world makes strictly true the prescriptions from the model-description even if the latter contains simplifications. As for the unauthorized game worlds, it seems that the actual world could be a part of them. But what is difficult with unauthorized games is that an agent is allowed to imagine anything and everything regarding the prop (the model-description here) and to interpret that prop in any way possible. Actually, this is the case when an agent imagines hippos starting from a painting which does not depict hippos. In the case of a (linguistic) description, this would amount to imagining propositions which would not have been prescribed by the description. But then, there is no scientific criterion any longer for selecting models. That is why a kind of link must be kept with the authorized game worlds (the "possible worlds" as I name them according to modal-logical terminology). If one digresses too far from them, one is likely to lose any notion of scientificity of models because agents become free to interpret the model description as they want. For example, in the framework of an unauthorized game, an agent could do "as if" a rugby ball was perfectly spherical, that is to say ascribe properties to an object in the target-system, even if it does not possess them (such as children can do as if a tree stump was dangerous).

To sum up, the work worlds make true the proposition according to which the actual tree stumps are bears according to the fiction, and no other proposition. In an authorized game world, an expansion of that description is allowed; in such worlds, this is true that stumps are bears,

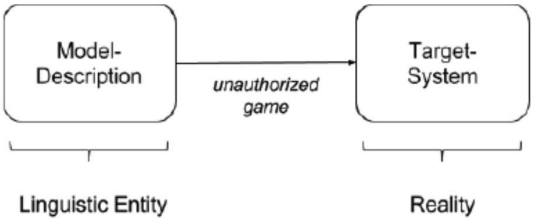

Figure 2: Toon's conception

that they are dangerous, that they growl, and so on (that list being longer or shorter according to the knowledge the agent has of bears). And in an unauthorized game world, according to Toon, such a proposition is true (in the actual world notably), but on the mode of "as if": what is true in the actual world is the statement *FPx* (with *F* a fiction-operator, *P* the predicate "being a bear" and where the value of x is an actual stump) and not directly the predication *Px* (this is literally false that actual tree stumps are bears). Yet, that does not provide any information directly linked to the actual world: the proposition is fictional in the actual world, that simply means that there are possible worlds compatible with the description and accessible from the actual world (in other words, it means that an actual object possibly has, according to the fiction, such and such properties in other worlds). Since the agent is in the actual world, the idea suggested by Toon consists in affirming that there are game worlds generated from that statement in an authorized way; those game worlds among which the actual world is not necessarily located, in the case of idealized properties for example. Writing $@ \vDash FPx$ only means that a fictional world w accessible from the actual world @ is such that $w \vDash Px$ (that is to say that the actual object which satisfies *FPx* in @ possesses a "counterpart" in a fictional world, which is accessible from @ and satisfies the predicate *P*). Furthermore, if a model-description was well and truly about an actual object, leaving Walton's works aside, how could one do "as if" an actual object had idealized properties, while every measure made concerns the properties that the actual object really possesses, and not the properties one ascribes to it by means of the imagination?[41] This issue will justify my analysis of properties in terms of causal powers.

41 If the degree of dangerousness of a tree stump could be measured, the result would be that the stump is not dangerous (unlike the bear).

One of the benefits of keeping an epistemological access to the worlds which perfectly exemplify the model-description (the only ones carrying weight in scientific representations) is to be able to rigorously study the behavior of a model across possible worlds. Besides, the analysis of the properties of fictional objects can be at the root of knowledge about actual counterparts of those objects (if any), but about those properties too. Indeed, if it is true in the actual world that $F(\exists x\ Px)$, which simply means that an actual description prescribes the existence of an object possessing the property *P* (in another world), Toon claims that, in the actual world, $\exists x\ FPx$, that is to say that there is an actual object about which it is fictional to say that it possesses the property *P*, since the aim of science is precisely to be about the actual world and its objects. However, as I have shown above, the fact that an object has the property *P* in the actual world in a fictional mode means that that object has counterparts which have P in fictional worlds. Then, by supposing that those fictional objects can be considered as counterparts of that actual object, a possible "retrograde" extrapolation consists in inferring certain pieces of information about the studied actual object, from knowledge obtained about its fictional counterparts. In particular, this requires to understand, on the one hand, the identity of objects across possible worlds and, on the other hand, the identity of a property sometimes imagined according to an authorized game, sometimes concretely exemplified. Whatever it be, I will look to explain that such an approach can be analyzed in terms of "lengthening" properties as world-lines.

4. Application of scientific models

Toon's works, like Frigg's, raise the issue of the comparison between models and target-systems. According to Walton, any comparison between two fictions leads to unauthorized games[42]; this may be more obvious when the comparison is made between a fictional object and an actual one.

Toon mentioned what Ronald Giere named "theoretical hypotheses", that is to say statements such as "the period of oscillation of the bob has

42 Walton, 1990, 410.

some value T0 and that it is fictional in our model that the bob oscillates with period T1, where T1 is within 10% of T0". However, it is difficult to understand what those comparative statements stand between, without imagining a fictional bob, as advocated by Toon; indeed, as I remarked, only the behavior of an actual bob is measurable. The presence of a fiction-operator ("in our model") in that statement leads to consider it as a prescription to imagine a fictional thing: this is the exact definition of an alternative possible world or even of a class of worlds.

Frigg also addressed the issue of the application of a scientific model considered as a fiction to a real target-system. Indeed, if the fictionalist approaches enable to understand the inner workings of models with simplifications, they generally do not clarify the issue of the success of those models. However, Frigg suggested an explanation based on the concept of property. The point is not to compare fictional objects in a model-description (or rather, as I suggest, in a multitude of possible worlds) with real objects in certain target-systems, since those objects are from different ontological classes. Then, Frigg proposed that a fictional object and a real one "possess certain relevant properties (…) and that these properties are similar in relevant ways"[43]. However, one is left with that notion of "similar" properties, yet this is problematic since, at this stage, that notion seems to be a kind of correspondence.

Abstract objects (like an ideal pendulum) do not have the same properties as concrete physical systems and Frigg approves of that idea and concedes that a property is not instantiated by a real object in the same way as by a fictional one. In the latter case, according to Frigg, in keeping with a game of make-believe, one has to imagine that the model object has properties, whereas, in the target system, one deals with real properties. In short, Frigg does not accept similarities between objects which are ontologically different, but bases his idea on similarities between properties which are ontologically different. One should not presuppose the notion of "similar properties", it has to be constructed or at least established.

43 Frigg, 2010b, 273.

To go beyond that philosophical issue, comparing purely local instances of properties amongst themselves is not enough. If possible worlds are representations[44] generated thanks to the model-description as a prop, one needs to be able to understand properties in a wider manner, by transcending the possible worlds. That proposal concerning properties is analogous to Hintikka's concerning individuals and consisting in reporting their identities in modal contexts. In first-order logic, an individual is an element of a domain. However, in order to explain that an element in a domain can be the "same" as an element in another domain, Hintikka[45] proposed to take another look at the notion of individual, by making it broader than a simple local element. In his view, an individual is a notional world-line across possible worlds. Formally, an individual is a function which, for each world, picks out a member from its domain. That element is the manifestation (or "embodiment") of that individual in this possible world. Thus, two elements a and b of domains of distinct worlds (such as w_1 et w_2) can be manifestations of the same individual I. For example, one can write: $I(w_1) = a$ et $I(w_2) = b$. In other words, a manifestation of an individual can be named "counterpart" for all the other manifestations of the same individual; in the example just above, a is a counterpart of b, and vice versa, since those objects (of distinct domains) are some manifestations of the same individual I.

In order to pursue Frigg's works, I suggest to define what the "similarity between properties" means. To do that, I consider properties as world-lines. A property is a function whose values are subsets of domains, made up of the elements that satisfy the property in the concerned world. In other words, a property φ which, for each possible world w, picks out the part of the domain of that world containing the objects that satisfy the property:

$$\varphi : w \longmapsto X \in \mathcal{P}(Dw)$$

with $\mathcal{P}(Dw)$ the set of the subsets of the domain of w. That conceptual tool allows me to clearly express the epistemological issues related to properties which play a role in games of make-believe. Indeed, I suggest

44 Speaking about "representations" here is appropriate since imagining a world which satisfies a description is indeed an authorized game in Walton's sense.

45 Hintikka, 1969, 137-138.

that one should understand those properties as world-lines as being epistemological constructions regarding scientific models. For example, in the framework of an authorized game, a model-description (which contains idealized properties) generates a whole class of fictional worlds whose domains contain objects that instantiate those properties perfectly. Those objects constitute the set-values of the property as a world-line. That approach still bears fictionalist traces, but at a level that is epistemologically different from the naive fictionalist approach: I reject every unique parasitic fictional entity (such as "the incomplete Ball"), but I accept, in the framework of modal logic, to consider that fictional (complete) possible worlds are rigorously compatible with a model-description (even it contains simplifications). As a consequence, it is necessary to explain the relations between fictional possible worlds and the actual world.

Nevertheless, I will not talk about the nature of those worlds; they result from an imaginative process, but I will not subscribe to modal realism for example. My approach is actually a modal one, more than a fictionalist one. Indeed, abstractions, notably, are not criteria for fictionality, but they are tools that enable models to be applied to the largest number of possible situations, in which the parameters that have been left undetermined by abstraction will vary from a situation to another. Concerning idealizations, and to come back to worlds-lines, strictly speaking, the value of a fictional idealized property in the actual world is the empty set. Indeed, by considering the possibility of a set-value of that property in the actual world, one begins to play an unauthorized game, as I explained. Otherwise, the actual objects have not been created to serve as props in such a game. Outside any epistemological framework, one seeks to apply an idealized model to an actual target-system. Lastly, I emphasize the importance of the fact that the values of properties are sets and not single elements of domains, as is initially the case with Hintikka regarding individuals: within the same situation, several objects can be considered as examples of the same property (for example during the study of the oscillations of two pendulums).

With the notion of properties as world-lines that I develop, I aim to continue Frigg's analysis which consists in comparing properties rather than objects. In Frigg's view, it seems clear that a model-description is

used according the *de dicto* modality of the fiction-operator (namely $F(\exists x\, Px)$ with P a definitional property of the description). Starting from that description, within authorized games, worlds in which $(\exists x\, Px)$ is true are generated (worlds in the plural and not just one model-system as Frigg proposed). Indeed, ascribing (idealized or not) properties to fictional entities is an authorized game; this is a p-representation according to Frigg's terminology. However, the t-representation cannot be called "representation" as I explained; this is a comparison at best (Frigg himself considers that the t-representation consists in a comparison by suggesting it is based on the similarity between properties). That comparison stands between properties of fictional objects in a model-system and properties of actual objects (by the way, that t-representation can bring to mind a search for an actual matching, like the one proposed by Toon, as I will explain, with a *de re* use of the fiction-operator regarding exemplifications of properties). Yet, in the case of science at least, the agents do not select the relevant properties themselves: it is the definitional properties described in the model-description which will be studied. But how can one compare an idealized property with a real one?

The crucial idea in my approach is precisely not to reason in terms of local properties (if not, inevitably, it leads to a correspondence issue between a property instantiated in an ideal world and another exemplified in the actual world). Just as an individual, in Hintikka's sense, can have manifestations with different properties, I suggest that a property can be possessed differently, notably by ontologically different objects. What is important is to be able to recognize the same property exemplified differently. Drawing properties as world-lines consists in constructing the set-values of those properties by making true their definitions given by the model-description.

I use that naturally modal notion of world-lines because, unlike Toon, I suggest that a model-description generates a multitude of possible worlds (since the *de dicto* modality of the fiction-operator makes it possible in an authorized way) and that the knowledge obtained from the study of ideal cases can be extrapolated to explain or predict certain actual phenomena. Indeed, according to Toon, a model-description is about actual objects or properties. However, he admits that an idealized property cannot be exemplified in the actual world (an idealized

proposition is false in the actual world when it is not in the scope of a fiction-operator), then he suggests to retain such a fiction-operator, but to use it according to the de re modality, in order to be coherent with his own approach (which reflects one of the aims of models which is to be about the actual world): in Toon's view, model-descriptions concern the actual world directly. Thus, Toon's proposal can be expressed as follows: ($\exists x\ FPx$) is true in the actual world.

Passing from the *de dicto* modality to the de re one can be understood as an unauthorized game, reflecting the scientific objective consisting in being about the actual world. But, as I explained, this just means that domains of possible worlds contain objects possessing the property P. Interpreting what Toon suggests in terms of modal logic amounts to affirming that there is at least a possible world w (accessible from the actual world @) where an object would possess the property *P* and that "that" object would exist in the actual world. However, Toon did not conduct the *de re* reading further. In order to develop it, I suppose that there is a link between fictional worlds and the actual world. A model-description (about a property *P*), within the framework of an authorized game whose prop is the property *P*, enables to write $@ \vDash F(\exists x\ Px)$ according to the de dicto use of the fiction-operator, that is to say that there is at least a world w accessible from the actual world @ and such that $w \vDash \exists x\ Px$ which amounts to considering that there is at least an element b of the domain of w such that $w \vDash Pb$. In terms of properties as world-lines, it means that $b \in \varphi(w)$ with φ the function related to the description of *P* by the model-description.

Passing to the *de re* modality of the fiction-operator consists in affirming that $@ \vDash \exists x\ FPx$. Here, the modal-logical analysis leads to a possible continuation of Toon's works. Indeed, $@ \vDash \exists x\ FPx$ if and only if there is at least an object a in the domain of the actual world such as @, $x{:=}a \vDash FPx$, namely if and only if every world w compatible with the fiction described in the model-description and accessible from @ is such that "$w,\ x{:=}a \vDash Px$". However, at this stage, that condition makes no sense insofar as the object a is an element of the actual domain, not one of the domain of *w*; those two distinct domains do not even contain objects of the same nature (real in @, but fictional in *w*). Thus, as I alluded to above, I use the notion of counterparts as being manifestations of the same individ-

ual. For at least an object *a*, @, $x{:=}a \vDash FPx$ if and only if, for every world w compatible with the fiction[46] and accessible from @, and for an object b in the domain of w, and such that there is an individual I with I(@) = *a* and *I(w) = b*, one obtains $w, x{:=}b \vDash Px$. To all appearances, one obtains the same result as in the previous paragraph according to the *de dicto* modality. But here, with the hypothesis supposing that there is such an individual, the object *b* in the domain of *w* is related to the actual object *a*, and this has influences on properties as world-lines. In other words, a hypothesis is formulated: the actual objects targeted by the existential quantification in the *de re* formula have counterparts in the authorized game worlds; for every world w among those game worlds, the counterparts of the targeted actual objects belong to the set $\varphi(w)$. The knowledge acquired concerning the property *P*, notably that about the construction of its set-values in different worlds, could be used to analyze its exemplifications by actual objects. In science at least, I suggest that a property as a world-line is drawn according to the dispositional profile of that property, namely by analyzing the manner in which it expresses itself (in terms of measure for example as I will explain below) in such and such circumstances. The counterfactual situations notably help to determine that, in such conditions, an object which has such causal powers can be recognized as an exemplification of the studied property (whose dispositions have been studied across fictional worlds generated by an authorized game). Thus, I agree with Frigg when he explained that the comparison between objects has to be carried out by the comparison of their properties. But I would add that the comparison is based on the causal powers of those local objects, namely on the dispositions of properties as world-lines.

To sum up my point of view, a model-description uses predicates related to certain properties. For example, the predicate "being perfectly spherical" is related to a property. But I suggest that that property has to be understood as a world-line whose set-values are subsets of domains of fictional worlds, and that, by extrapolating those lines, there are also actual objects which satisfy that property (which scientists suppose when

46 By definition, these worlds compatible with the fiction seem to precisely correspond to what Walton called authorized game worlds.

they look to apply an idealized model to an actual situation). That property as a world-line can be perfectly instantiated (in possible worlds generated by an authorized use of the model-description, that is to say in the authorized game worlds in Walton's sense) or can be approximatively exemplified (in the actual world notably). In the latter case, it is an unauthorized game insofar as one looks to non-literally interpret the model-description: indeed, one supposes that the idealization performed with the term "perfectly" is not just used to target fictional worlds, but serves to target the largest possible number of actual situations too.

An unauthorized game consists in imagining that counterparts of actual objects have the property *P*. An extrapolation (according to the *de re* modality of the fiction-operator) consists in supposing that actual objects belong to the set $\varphi(@)$. In other words, from an ontological point of view, $\varphi(@)$ is the empty set, whereas from an epistemological point of view, $\varphi(@)$ is the set composed by the actual objects whose (following the example above) degree of sphericity is satisfactory according to the model. Such satisfaction is allowed when the dispositional profiles of those objects are similar to those of the fictional objects, namely the dispositions described by the model. The comparison between dispositional profiles consists in counterfactual examinations (based on tests of measurements or on abstract reasonings for example). That comparison is crucial in the explanation of the successful applications of scientific models. If the measurements of the causal powers of an actual object are similar to those described in the model (namely identical but in taking into account the relative error defined by the model), then that object exemplifies the studied property. In other words, two objects are exemplifications of the same property if they have the same behavior in the same circumstances. The predictions emitted about one of those objects can be extended to the other one. For example, in idealized cases, the equation for the motion of a perfectly spherical ball is rigorously respected, while a relative error occurs during the study of the motion of a ball in the actual world. If that error is in the interval whose endpoints have been set by the model, then the studied actual objects are members of the set-value $\varphi(@)$. Indeed, most often, the model itself contains the definition of those tolerated limits regarding the measurements (as with "theoretical hypotheses" as Giere named them); those

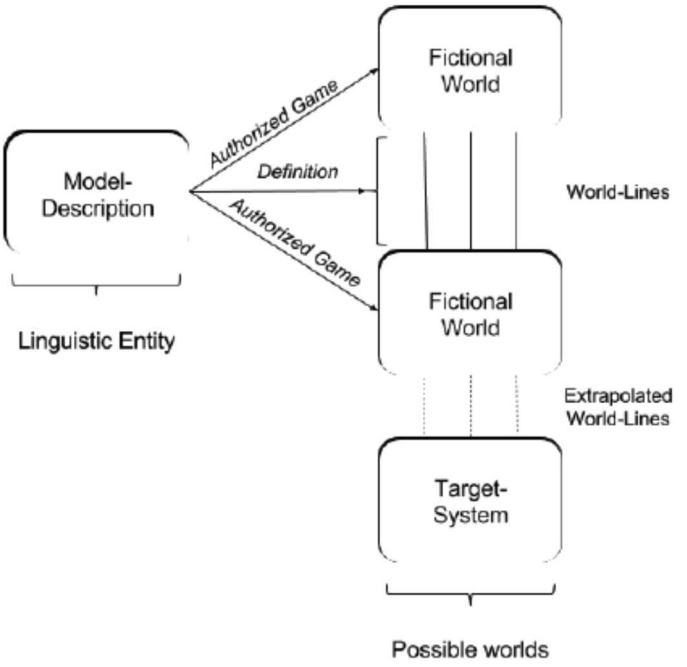

Figure 3: Extrapolation of properties as world-lines

hypotheses are guidelines used to recognize the set-values of those properties as world-lines, according to the causal powers of their supposed elements. That kind of rules of generation will exert influence on the size of the set-value of a property in the actual world. For example, according to a given model a basketball can be considered as being comparable to a perfectly spherical object, while, according to another more rigorous model, it will not be the case. Thus, a property as a world-line is always relative to a given model since its set-values are constructed according to the relative error defined within that model.

The purpose of science is to be about the actual world. However, there is no commitment concerning the actual world with a *de dicto* use of fiction-operators. I agree with Toon on acting according to the de re modality (even if I emphasize that it is an unauthorized game), but I suggest that it is not enough. Indeed, such an unauthorized game does not teach us anything or not much, without supposing that the generated fictional objects (which rigorously satisfy the definitional properties) are

counterparts of actual objects (targeted by the existential quantification in the *de re* formula). Only then can scientists carry out a "retrograde" extrapolation consisting in inferring information about actual objects starting from the acquired knowledge about their fictional counterparts. More precisely, that extrapolation consists in inferring knowledge about those actual objects which exemplify certain properties from information concerning their counterparts which instantiate those same properties in the authorized game worlds. For example, if, according to a model, "every object exemplifying the property *P* also exemplifies the property *Q*", then that statement is only true about objects which rigorously are *P*, namely only about fictional objects in the game worlds (if *P* is idealized, for example). But those objects are counterparts of actual objects, so one can infer that those actual objects are also *Q*, by extrapolation of the world-line of the property *Q*. This will be confirmed by experimental verification consisting in measuring the causal powers of the actual objects and by assessing whether the results are within the relative error determined in the model. Thus, even if that aspect concerning the causal powers as criteria in the construction of properties as world-lines will have to be improved, it is already notable that the cardinality of an actual set-value of a property (expressed in an idealized way within the model-description) depends on the relative error tolerated during the application of that model; the more rigorous a model is, the smaller the actual set-value of the studied definitional property will be (compared to a more flexible model whose degree of uncertainty allows for more actual objects to exemplify that property). Thus, the comparison between the dispositional profiles of the properties will be based on that kind of criteria too.

According to Walton, a model-description involving idealizations would be satisfied by a class of work worlds, and generate both classes of authorized and unauthorized game worlds. In my view, the authorized game worlds (which are complete and make the model-description strictly true) are the only relevant ones in philosophy of science.

I have proposed a possible reinterpretation of Frigg's and Toon's perspectives. An unauthorized game, related to the *de re* modality of the fiction-operator, consists in supposing that an actual object possesses counterparts in the game worlds generated from the model-description

in an authorized way. Such a game is a kind of unauthorized access to the authorized game worlds. For example, Arthur Conan Doyle's work, considered as a prop, generates worlds compatible with that fiction. The unauthorized game consists in affirming that there exists an actual person who possesses counterparts in those game worlds. Finally, an extrapolation consists in affirming that properties which are instantiated in fictional worlds are exemplified in the actual world; in other words, that actual objects possess the properties instantiated by their counterparts. This amounts to "extending" properties as world-lines from authorized game worlds to actual situations.

5. Conclusion

Properties as world-lines are useful to illustrate the mode of construction of scientific properties according to a perspective compatible with modal logic. Studying those lines enables me to pursue the works related to the analogy between models and games of make-believe. Indeed, I explained that the similarity between properties suggested by Frigg is too vague a concept to be primary in philosophy of science, and I suggested constructing that similarity in terms of causal powers, regarding the dispositional profiles of those properties. Furthermore, in analyzing Toon's proposal (consisting in a de re use of the fiction-operator) from a modal-logical perspective, I have explained that it concerns only the accessible worlds from the actual world (and not the actual world itself). Inferring information about the actual world requires a certain kind of extrapolation, which I suggested doing by means of world-lines. The ascription of a property instantiated in an idealized way in a possible world to an actual object of a target situation is an extrapolation consisting in extending (in a conceptual sense) properties as world-lines. Indeed, that leads us beyond Walton's work. Since I cannot rigorously brand that relation in terms of games of make-believe, then I suggest resorting to epistemological extrapolations consisting in extending a (possibly idealized) property as a world-line up to the actual world.

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