Understanding Admissibility

George Masterton
(Philosophy Department, Lund University)
george.masterton@fil.lu.se

Introducing Admissibility

Chance is an enigmatic topic of philosophical interest; consequently, there are many strong opinions on the subject and very little agreement. That said, there is one point upon which all are agreed; whatever chance is, if the chance for *A* is known to be *x*, then it is *prima facie* reasonable to believe *A* to degree *x* and to act/bet accordingly. If you know the coin is fair, then you should be as prepared to bet heads as tails; if you know that it is biased so that the chance of heads is 2/3, then you should be prepared to bet on heads at odds of 2:1. There are almost as many notational variants of this principle as there are people who have written on the topic of chance, but for our purposes van Fraassen's is instructive.

> **Miller's Principle:** My subjective probability that *A* is the case, on the supposition that the objective chance [at *t*] of *A* equals *x*, equals *x*.
> Symbolically: $[C](A|ch_t(A) = x) = x$.[1]

van Fraassen's name for this principle is a little misleading; Miller's principle is actually a principle that relates probabilities at different linguistic levels. Miller's principle, roughly stated, is: let P_1 be a probability function defined on the object language, *A* be a sentence of that object language, *x* be a real on the unit interval and P_1 be a probability function defined on the meta-language; then $P_2(A|P_1(A) = x) = x$. Hence van Fraassen's Principle is a specific application of Miller's Principle – with $C = P_2$ and $ch_t = P_1$ – and not that principle in its full generality. In any case, van Fraassen took this principle to answer his 'how' question.

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[1] Van Fraassen, 1989, 82.

> … I stated the fundamental question about objective chance: why and how should it constrain rational expectation? The 'how' is answered by Miller's Principle and its generalizations.[2]

Among those 'generalizations' that also answer his 'how' question, van Fraassen might have included Lewis' Principal Principle (PP).

> **Lewis' (PP):** Let *C* be any reasonable initial credence function. Let *t* be any time. Let *x* be any real number in the unit interval. Let $[X_t^A]$ be the proposition that the chance, at time *t*, of *A*'s holding equals *x*. Let *E* be any proposition compatible with $[X_t^A]$ that is admissible at time *t*.
> Then $C(A|[X_t^A]E) = x$. [3]

The two principles are more or less[4] identical save for the inclusion of admissible *E* in Lewis' PP. There are two justifications for admissible *E*'s inclusion: one that is generally accepted and another that is peculiar to those working within the Lewisian program on objective chance. The generally accepted justification is that the plausibility of reasonable credence tracking chances is thought to increase where such tracking is largely invariant to further conditionalization. For instance, if you know that the chance of heads on the next toss is 1/2, then your credence in the next toss landing heads should be 1/2 and should remain so even once you have found out that the coin is a 2 euro coin, that the temperature is 30 Celcius, etc. The other justification is that Lewis' RPP (see below) cannot be derived from the PP unless history and natural laws are (generally) admissible$_t$. Lewis, and those following his lead, rely on the derivation of the RPP from the PP to justify the former; thus admissibility is important in the justification of Lewis' RPP.

> **RPP:** Let *C* be any reasonable initial credence function. [Let $H_{tw}T_w$ be that proposition that holds at all and only those worlds historically and nomologically possible relative to *w* at *t*]. Then for any time *t*, world *w*, and proposition *A* in the domain of P_{tw}; $P_{tw}(A) = C(A|H_{tw}T_w)$.

Why is it important for Lewis that the RPP is justified? Lewis originally says of the RPP that it has the 'form of an analysis' of chance.[5] Later Lewis held this principle to state the definitive 'role' that something must satisfy if it is to count as objective chance.[6] I have argued elsewhere that this, together with

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[2] Van Fraassen, 1989, 195.
[3] Lewis, 1980, 87.
[4] This qualification is needed as the Principal Principle is also restricted to initial reasonable credence functions, whereas "Miller's" Principle applies more generally to all reasonable credence functions
[5] Lewis, 1980.
[6] Lewis, 1994.

Lewis' Canberra Planer predilections, is enough to convince ourselves that Lewis held the RPP to be, or at least to motivate, an analysis of objective chance in terms of reasonable credence conditional on prevailing history and natural laws. [7] For Lewis, that such an analysis could be more or less derived from such an uncontroversial fact about chances as his PP was a boon. The same holds for many who are tempted by the *chance as ultimate belief* thesis[8]: the thesis that objective chances$_t$ are objective degrees of belief conditioned upon some ultimate evidence$_t$; in Lewis' case, prevailing laws and history$_t$. But this derivation[9] is only (generally) valid where $H_{tw}T_w$ is (generally) admissible $_{tw}$ [10]; one cannot derive the RPP from what van Fraassen refers to as Miller's Principle. Consequently, if a Lewisian wishes to justify an analysis of chance in terms of that belief that is reasonable given our ultimate evidence on the basis of credence's conformity to chances, then they need the concept of admissibility.

Unfortunately, Lewis explicitly failed to rigorously define admissibility; settling instead for the following rough and ready characterization:

> Admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about chances of those outcomes.[11]

Despite this inauspicious start, headway was made through the identification of two sufficiency conditions[12]:

1. *E* is 'as a rule'[13] admissible at time *t* (admissible$_t$) if *E* pertains entirely to times earlier than, or including, *t.*

2. E is admissible if it is a history-to-chance conditional.

He later allowed that the axioms and theorems of optimal systematizations of collections of history-to-chance conditionals (A.K.A. natural laws) are also

[7] Masterton, 2010.
[8] Williamson, 2008.
[9] Lewis,1980.
[10] I have found that writing on this and related topics is aided by a judicious use of subscripts for there is much indexing to worlds and times. Hence I use 'admissibletw' as an abbreviation of 'admissible at time t and world w' and likewise for other indexical concepts.
[11] Lewis, 1980, 94.
[12] Lewis, 1980, 92.
[13] This caveat covers such eventualities as the testimony of time travellers or infallible soothsayers. Their testimony before time t is part of our historyt, but as conditionalizing upon it must break the link between reasonable credence and chancet, so such historict occurrences must be inadmissiblet.

admissible.[14] With these sufficiency conditions he could rule $H_{tw}T_w$ (generally) admissible, and thereby, derive the RPP from the PP.

It took fourteen years before some of the many flaws in this initial characterization of admissibility were addressed. Lewis[15] – after prompting by Thau[16] – finally made two amendments to his concept of admissibility, together with one to his Principal Principle. The first amendment was to allow that admissibility admits of degrees: that a proposition can be more or less admissible.

> Admissibility admits of degree. A proposition *E* may be imperfectly admissible because it reveals something or other about future history; and yet it may be very nearly admissible, because it reveals so little as to make a negligible impact on rational credence.[17]

He then weakened the PP so it applies where *E* is admissible or 'nearly' so. Finally, his most important amendment was to agree with Thau[18] that admissibility$_t$ is relative: one proposition is admissible$_t$ for another, not admissible$_t$ *tout court*.

> [D]egrees of admissibility are a relative matter. The imperfectly admissible *E* may carry lots of inadmissible information that is relevant to whether *B*, but very little that is relevant to whether *A*.[19]

These are certainly substantial improvements, yet still there is no necessary and sufficient condition for admissibility. What we do now have is a fairly clear idea of some of the main features of the concept:

- Admissibility is indexical: there is an admissibility for every time, and possibly also for every world.
- Admissibility is relative: one proposition is admissible for another.
- Admissibility admits of degree: one proposition can be more or less admissible for another.
- A proposition is generally admissible$_t$ iff it is admissible$_t$ for every proposition for which a chance$_t$ is defined.
- Propositions that hold of the world prior to *t* are generally admissible$_t$ as a rule, and natural laws are generally admissible without exception.

Like a golden thread running through this list is Lewis' original characterization of admissible propositions as 'the sort of information whose

[14] Lewis, 1994.
[15] Lewis, 1994.
[16] Thau, 1994.
[17] Lewis, 1994, 486.
[18] Thau, 1994, 500.
[19] Lewis, 1994, 486.

impact on credence about outcomes comes entirely by way of credence about chances of those outcomes.

Other perspectives on admissibility

Other commentators have more or less followed Lewis' lead on admissibility. For instance, Bigelow[20] gave the following characterization of the concept immediately before Thau and Lewis' later amendments and one can see that they adhered closely to the then established view.

A proposition will be admissible [at *t*] iff it does not covertly smuggle in information about the future, information which, since it is about the future, might bear on the present$_t$ rational credence about outcomes in a way that short-circuits the normal route via the present rational credence about present chances of outcomes.

Then came Thau's insight that though Lewis' earlier characterization of admissibility was essentially correct, it missed the essential feature that admissibility is always relative to another proposition.

> A proposition is admissible [at *t*] if it doesn't provide direct information about the outcomes of chancy events that occur subsequently to *t*. [...] A proposition is inadmissible with respect to another proposition if it provides direct evidence about it. [21]

As I have already stated, Lewis was so impressed with this insight that he immediately adopted it.

Around the same time, Hall[22] noted that a necessary condition for *E*'s *general* admissibility$_t$ is that either the chance$_t$ of *E* is undefined, or else it is 1. Halpin[23] concurred with this opinion shortly thereafter. The argument for this condition is simple: If the chance$_t$ of *E* is defined, then *E*'s general admissibility$_t$ requires *E* be admissible$_t$ for itself; and so $C(E|E, X_t^A) = x$. But this is only so for reasonable *C* if $x = 1$; consequently, either the chance$_t$ of generally admissible$_t$ *E* is undefined, or its chance$_t$ is 1. This is an important result as the general validity of the derivation of the RPP from Lewis' PP depends upon the general admissibility$_{tw}$ of prevailing$_{tw}$ history and natural law. Therefore, either historic$_{tw}$ propositions and natural laws$_w$ have a chance$_{tw}$ of 1, or their chance$_{tw}$ is undefined, if the RPP is to be derivable from

[20] Bigelow, 1993, 454.
[21] Thau, 1994, 493, 500.
[22] Hall, 1994.
[23] Halpin, 1998.

the PP. This is an important lesson in the context of the debate on Humean Supervenience; through the PP, reasonable credence constrains the chances of generally admissible propositions to trivial values. This places us on the horns of a dilemma: either we accept that some chances can be dictated by what reasonable credence allows, or we introduce chance gaps so that, at the very least, no chance is ever defined for generally admissible propositions. Often people have no problem accepting that historical propositions have trivial chances, but which choice one makes for natural laws is another matter entirely[24].

In the last decade or so there have been attempts to give a full definition of admissibility by Loewer, Hall and Hoefer. Hall's attempt is the most interesting and distinct of these. [25]

> *E* is admissible with respect to [initial reasonable] credence $C_{[\,]}$, proposition *A*, and time *t* iff $C_{[\,]}$ takes it as certain that the *t*-chances treat *A* and *E* as independent.
> Symbolically the "definiens" is:
> $$C\big(ch_t(A|E) = ch_t(A)\big) = 1.$$

To his credit, Hall's definition is formal and precise. Moreover, the definiens can be demonstrated to be a sufficient condition for the Principal Principle to apply; both from the premises Hall assumes and from the assumption that reasonable C conforms to $C(A|X_t^A) = x$. The latter demonstration builds on a rather tricky proof, originally by Skyrms (1988), where one establishes that any reasonable credence C that obeys $C(A|X_t^A) = C(A|ch_t(A) = x) = x$, must also obey $C\big(A\big|E, X_t^{A|E}\big) = C(A|E, ch_t(A|E) = x) = x$. But if C obeys the later and $C\big(ch_t(A|E) = ch_t(A)\big) = 1$, then it will obey $C(A|E, X_t^A) = C(A|E, ch_t(A) = x) = x$., which is the PP. Hence, $C\big(ch_t(A|E) = ch_t(A)\big) = 1$ is a sufficient condition for the PP to apply for any C that respects $C(A|X_t^A) = x$. A final advantage of Hall's definition is that it is fairly obvious how it could be used to generate a definition of degree of admissibility.

[24] Though it is way beyond the scope of this paper, it turns out that the two solutions that have been offered to the greatest challenge facing Humean Supervenience, commonly known as The Bug, correspond to these alternatives. Lewis (1994)/Hall's (1994) response corresponds to restricting the chances for prevailing laws to unity and accepting the uncomfortable consequences for chances that follow from this, whilst Hoefer (2007) has offered a chance gap solution that corresponds to the latter alternative.

[25] Hall, 2004, 102.

> *E* is admissible to degree α with respect to [initial reasonable] credence $C_{[\,]}$, proposition *A*, and time *t* iff $C_{[\,]}$ takes it as certain that $1 - |ch_t(A|E) = ch_t(A)| = \alpha$.

However, that $C\big(ch_t(A|E) = ch_t(A)\big) = 1$ is sufficient for the PP to apply where C is reasonable is not enough to establish this condition as a definiens for admissibility. Moreover, admissibility defined in the manner outlined by Hall would be very different from how admissibility is typically conceived; Hall's definition has little to do with the manner in which one proposition informs on another via its chance. Both of these points indicate that $C\big(ch_t(A|E) = ch_t(A)\big) = 1$ may be unsuited to the task of defining admissibility. A further potential problem is that the degree of admissibility definition I extrapolated from Hall's definition of relative admissibility does not behave as we might expect. Consider a seer's testimonies on the results of a soon to be conducted toss of a fair die and a fair coin in a C that grants that the seer is infallible. According to that definition, the seer's testimony that heads will be the result of the coin toss is 3 times as admissible $\left(\alpha = \frac{1}{2}\right)$ as that same seer's testimony that the result of the die cast will be 5 $\left(\alpha = \frac{1}{6}\right)$ in such a C. But in both cases the reason for the seer's testimony's inadmissibility – namely, the granted entailment of the result in question by that testimony – is the same, so it is natural to expect the two testimonies to be equally inadmissible for their respective results. That they are not is a bit of an unwelcome surprise. True, the extrapolated degree of admissibility definition is not Hall's, and true, one might reconcile oneself to degrees of admissibility that depend on the chances involved, but still I take this as grist for the mill.

Hoefer's concern lay mainly with general admissibility, which he refers to simply as ‘admissibility’. He offered two definitions of this concept:

> Any proposition (your “evidence”) that does not contain information relevant to the outcomes of chance events except by containing information about their (objective) chances [is admissible]. [26]

> Propositions that are admissible with respect to outcome-specifying propositions A_i contain only the sort of information whose impact on reasonable credence about outcomes A_i, if any, comes entirely by way of impact on credence about the chances of those outcomes. [27]

[26] Hoefer, 1997, 324.
[27] Hoefer, 2007, 553.

The latter definition is of particular interest as it plainly contains within it a definition of relative admissibility; one that seems to be exactly like that proffered by Loewer. [28]

Loewer's definition, and the sufficiency condition for inadmissibility – which may as well have been given as a definition – that Loewer[29] draws from it, strike me as the best characterizations of the concept available to date.

> *Q* is admissible relative to *A* at time *t* iff *Q* provides information about *A* only by providing information about the chance of *A* at *t*. [30]

> Information about A is inadmissible if it is information about A over and above information about A's chance. [31]

Loewer's definitions of (in)admissibility capture the relative and indexical nature of the concept whilst at the same time incorporating Lewis' original specification in a succinct and tidy way. I particularly like the later 2004 condition for inadmissibility, which seems to me to capture everything currently understood about the concept in one concise sentence.

All that having been said, there is still substantial room for improvement. When is information about *A* information over and above information about *A*'s chance? How do we quantify the degree to which *A* informs over and above informing about *A*'s chance? At best, the culmination of 30 years of ruminating on this concept have provided us with a promissory note of a definition; one in which a great many details are still to be filled in and made precise. I now turn to the task of answering these outstanding concerns.

A framework for understanding admissibility

Our task here is to give a formal intensional definition of relative admissibility and its degrees. There are several desiderata against which such a definition might be judged and these can often be in tension. The following is a partial list of these desiderata:

Continuity: Ideally, a rigorous definition of an established concept should be as faithful as possible to its informal characterisations. Of course, if there is fundamental disagreement between these informal

[28] Loewer, 2001, endnote 5.
[29] Loewer, 2004, 1116.
[30] Loewer, 2001, endnote 5.
[31] Loewer, 2004, 1116.

characterisations, then this desideratum can only ever be partially satisfied.

Improvement: A new definition of an established concept should be an improvement on those that have been offered previously. The improvements sought herein are in terms of clarity, precision and quantifiability, though others might also be pertinent.

Informativity: An intensional definition that makes the extension of a term epistemically transparent is to be preferred to one that leaves such epistemically opaque.

Clarificatory: Where the extension of a term has been in dispute, it is a virtue of a definition if it reveals such disagreement to be rational or, otherwise, explicable.

Coherence: Any definition must be logically consistent.

Because these, and other, desiderata might be conflicting, definitions have to be judged in terms of the balance they strike between them. Unfortunately, commentators are likely to value the desiderata differently, differ in terms of how they measure definitions against them and even differ in terms of how they balance them. This all makes it very difficult, if not impossible, to propose a definition that will meet with every commentator's approval.

That having been said, it can be worthwhile to try and give a precise, intensional definition of a disputed concept in order to clarify the terms of the dispute. In this spirit I shall suggest a basic framework for constructing definitions of relative admissibility and its degrees based on the probabilistic notions of conditional independence and resiliency. Conditional independence is typically[32] defined as follows:

> If $P(A|B,C) = P(A|C)$ [where $P(B,C) > 0$], we say that *A* and *B* are *conditionally independent* given *C*; that is, once we know *C*, learning *B* would not change our belief in *A*. (Pearl, 2000, p.3).

Now consider the following substitution instance of the above.

> If $C(A|E,X_t^A) = C(A|X_t^A)$ [where $C(E,X_t^A) > 0$], we say that *A* and *E* are *conditionally independent* given X_t^A; that is, once we know X_t^A, learning *E* would not change our belief in *A*.

[32] The gloss given to the definition after the semi-colon is typical but might not be universally endorsed; some might argue that conditional independence is not equivalent to knowing C making credence in A invariant to learning B. For my purpose – conceptual analysis of Lewis' admissibility – it suffices that this gloss of screening off is common in the literature and that the connection between conditional independence and indirect informing is widely acknowledged.

Conditional independence is often known as *screening off*: i.e., X_t^A screens off *A* from *E* in *C* exactly where *A* and *E* are *conditionally independent* given X_t^A in *C*. Learning *E* after X_t^A would not change our belief in *A* iff *E* only informs on *A* via X_t^A, if at all. In the main (Lewis/Loewer) tradition on relative admissibility, *E* is admissible$_t$ for *A* iff *E* informs on *A* only via *A*'s chance$_t$. All this suggests that relative admissibility might be fruitfully defined in terms of screening off by chances in something like the following manner:

E* is admissible$_t$ for *A iff $C(A|E,X_t^A) = C(A|X_t^A)$ [where $C(E,X_t^A) > 0$].

We can simplify this proposal by noting that reasonable credence functions are regular according to Lewis (1980). Such regularity implies that $C(E,X_t^A) = 0$ if, and only if, *E* and X_t^A are inconsistent/incompatible. Where *E* and X_t^A are incompatible the Principal Principle does not apply (see earlier citation) and the question of *E*'s admissibility$_t$ for *A* is mute. It follows that the question of *E*'s admissibility$_t$ for *A* is only pertinent if $C(E,X_t^A) > 0$. Consequently, in any definition of admissibility in terms of conditional independence in initially reasonable C, the "where" clause above will be redundant and may be omitted giving:

E* is admissible$_t$ for *A iff $C(A|E,X_t^A) = C(A|X_t^A)$.

Recall that this is only a framework, and not a definition *per se*; many details still have to be resolved before the above can spawn a definition. Despite these flaws the above does already have some merits. Firstly, the definiens is precise and familiar to those with a working knowledge of probability theory. Secondly, there is a continuity between this framework and the definitions offered by Loewer, Lewis, Hoeffer, etc through the oft assumed association between 'C screening off *A* from *B*' and '*B* informing on *A* only via C'. Thirdly, if admissibility is defined in terms of screening off, then degrees of admissibility are naturally defined in terms of degrees of screening off. Skyrms (1977) introduced *resiliency* as a measure of the degree to which one proposition screens off another from a third, so all we need do in order to define a measure of admissibility$_t$ is to co-opt Skyrms' notion of resiliency:

***E* is admissible at *t* for *A* to degree α** iff

$$1 - |C(A|E,X_t^A) - C(A|X_t^A)| = \alpha,$$

equivalently:

E is admissible at t for A to degree α iff credence in A, given the chance$_t$ of A, has resiliency over E of α.

It naturally follows within this framework that E is admissible$_t$ for A iff E is admissible at t for A to degree 1; or, to paraphrase Skyrms (1977):

> To say that [X_t^A] *shields-off* [A] from [E] is to say that [$C(A|X_t^A)$] has resiliency 1 over *E.*

All of this is promising, but we have yet to produce a definition. This becomes clear when we attempt to formalize the above framework as a definition schema. Strictly speaking the "definition" and "principles" offered in this text, and more generally in the literature as a whole, are not definitions and principles *per se* but rather definition and principle *schemata*: i.e. representations of sets of related definitions and principles. Other notable examples of such schemata are Tarski's T-schema, the standard definition of conditional probability, $A \rightarrow (A \vee B)$ of propositional logic, etc. A schema, generally, is a system consisting of two parts: a *schema-template* and a *side note*. The former can be thought of as being comprised of three types of component: schematic constants, schematic variables and quantifier-bound variables. It is important to recognize that the schematic variables of a schema-template and its quantifier-bound variables are very different: schematic variables range over formulas whereas quantifier-bound variables range over objects in the universe of discourse. For instance, the universal instantiation schema – $\forall x[A(x) \rightarrow A_a^x]$, where A is a formula of a first-order propositional language, x is a variable, a is a term substitutable for x in A, and A_a^x is the formula obtained once a has been substituted for x in $A(x)$ – would make little sense unless there was a distinction between the schematic variables A and a, the schematic constants $\forall$ and $\rightarrow$ and the quantifier-bound variable x.

While a schema-template is a purely syntactic object – a string-type with string-tokens for every permutation of the schematic variables – its attendant side note expresses a proposition. This proposition determines the appropriate interpretation of the instances of the schema in question; the proposition – definitions, principles, axioms, etc – expressed by the schema instances. To this end, the side note gives the domains for the schematic variables, tells us how to read the schematic constants, and gives the intended domains of any quantifier-bound variables present.

Returning to the proto-definition of relative admissibility offered earlier we have the proto-schema template:

E* is admissible$_t$ for *A iff

$$C(A|E, X_t^A) = C(A|X_t^A).$$

This proto-template is meaningless without an attendant side note identifying the schematic constants and variables and quantifier bound variables, as well as the relevant domains of the latter. Plainly, the language of the schema instances is English and the schematic constants are ‘is’, ‘admissible’, ‘for’, ‘iff’’, ‘(‘, ‘|’, ‘)’ and ‘=’. Equally plainly, *E*, *A* and *t* are schematic variables ranging over designations of propositions and times, respectively. But how are C and X_t^A to be read in the schema? Obviously, they are not constants of the schema, so they are either quantifier bound variables – where the quantifiers have yet to be added to the schema template – or else they are variables of the schema. If C is a schematic variable, then it will range over designations of reasonable initial credence functions, if it is quantifier bound in the schema then its intended domain will be the reasonable initial credence functions. If X_t^A is a schematic variable, then it will range over designations of chance$_t$ of *A* propositions; if quantifier bound, then its intended domain will be the chance of *A* at *t* propositions.

We can quickly exclude three definition schemata if we consider the four schemata templates where C and X_t^A are quantifier bound.

1. E* is admissible$_t$ for *A iff $\forall C \forall X_t^A [C(A|E, X_t^A) = C(A|X_t^A)]$.

2. E* is admissible$_t$ for *A iff $\forall C \exists X_t^A [C(A|E, X_t^A) = C(A|X_t^A)]$.

3. E* is admissible$_t$ for *A iff $\exists C \forall X_t^A [C(A|E, X_t^A) = C(A|X_t^A)]$.

4. E* is admissible$_t$ for *A iff $\exists C \exists X_t^A [C(A|E, X_t^A) = C(A|X_t^A)]$.

All these templates have the same side note; namely: The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the intended domain of X_t^A is the chance$_t$ of *A* propositions – $\{ch_t(A) = x : x \in [0,1]\}$ –, and the schematic variables are *E* and *A* – ranging over designations of propositions – and *t* – ranging over designations of times.

The third and fourth such definition schemata are obviously too weak, as it surely cannot suffice that *E* is screened off from *A* in a single credence function by a (all) chance$_t$ of *A* proposition(s) for it to be admissible$_t$ for *A*. We can also rule out the second. As $C(A|E, X_t^A)$ is either greater than, or equal to, $C(A|X_t^A)$ when the latter is equal to zero and less than, or equal to, $C(A|X_t^A)$

when the latter is equal to 1, so there must exist an X_t^A such that $C(A|E, X_t^A) = C(A|X_t^A)$ in every C. Hence the second schema implies that everything is admissible for everything else and can be rejected on this account. Indeed, all schemata conforming to the framework where X_t^A is existentially quantified can be ruled out on this account.

So far we have only one viable definition schema for relative admissibility, but what about all those schemata where C or X_t^A are schematic variables as opposed to quantifier bound variables? Here we encounter a problem: As a definition schema represents a set of definitions – one for every permutation of the values of schematic variables –, so it follows that, if a schematic variable occurs in the definiens of a schema template but not in the definiendum, then there will be multiple definitions for the same definiendum. This can cause problems for, unless the definiens of each of these multiple definitions of the same definiendum are equivalent, such a schema will be incoherent. As a general rule, it is best to avoid such problems by ensuring that any schematic variable occurring in the definiens of a template also occurs in the definiendum, and vice versa. This assures a one to one correspondence of definiendum to definiens in the instances of the schema. Where C is concerned, this is the appropriate route to take. Maintaining X_t^A as a quantifier bound variable this gives another definition schema for relative admissibility:

5. E* is admissible$_t$ for *A* in *C iff $\forall X_t^A\ [C(A|E, X_t^A) = C(A|X_t^A)]$.

The language of this schema's instances is English, the intended domain of X_t^A is the chance$_t$ of *A* propositions – $\{ch_t(A) = x\colon x \in [0,1]\}$ –, and the schematic variables are *E* and *A* – ranging over designations of propositions –, C– ranging over designations of reasonable initial credence functions – and *t* – ranging over designations of times.

Applying the same method we can construct two further schemata where X_t^A is a schematic variable.

***6. E* is admissible for A with respect to X_t^A** iff $\forall C\ [C(A|E, X_t^A) = C(A|X_t^A)]$.

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the schematic variables are *E* and *A* – ranging over designations of propositions – and X_t^A–ranging over designations of chance$_t$ of *A* propositions.

***7. E* is admissible for *A* in *C* with respect to X_t^A** iff $C(A|E, X_t^A) = C(A|X_t^A)$.

The language of this schema's instances is English, the schematic variables are *E* and *A* – ranging over designations of propositions –, C – ranging over designations of reasonable initial credence functions – and X_t^A –ranging over designations of chance$_t$ of *A* propositions.

The problem with this approach is that relative admissibility so defined is chance relative: there being a relative admissibility of *E* for *A* for every logically possible chance$_t$ of *A*. To this author's mind, and contrary to Meacham (2010), such chance relative admissibility is not sufficiently continuous with the literature on the subject to be acceptable. However, simply deleting 'with respect to X_t^A' from the definiendum in the above templates leads to the aforementioned problem of multiple non-equivalent definitions for one and the same definiendum. For example, two instances of schema 6 with 'with respect to X_t^A' deleted from the definiendum would be.

E* is admissible$_t$ for *A iff $\forall C[C(A|E, ch_t(A) = 0) = C(A|ch_t(A) = 0)]$.

E* is admissible$_t$ for *A iff $\forall C[C(A|E, ch_t(A) = 1) = C(A|ch_t(A) = 1)]$.

This implies that $\forall C[C(A|E, ch_t(A) = 0) = C(A|ch_t(A) = 0)]$ iff $\forall C\ [C(A|E, ch_t(A) = 1) = C(A|ch_t(A) = 1)]$. While there are values of *E* and *A* that satisfy this equivalence, there are also plenty that do not.

To get around this problem one can add a condition to the side note to ensure that the schematic variable X_t^A ranges over designations of a single chance$_t$ of *A* proposition. This move will make admissibility implicitly relative to some particular chance$_t$ of *A*, but to which chance$_t$ of *A* should it be so relative? There are many choices one could make here and which one feels appropriate seems to be more a matter of taste than anything else; indeed, any choice seems arbitrary. In any case, examples of the creed include:

8. E* is admissible$_t$ for *A iff $\forall C[C(A|E, X_t^A) = C(A|X_t^A)]$.

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the schematic variables are *E* and *A* – ranging over designations of propositions – and X_t^A – ranging over designations of the proposition giving the actual chance$_t$ of *A*.

9. E* is admissible$_t$ for *A* in *C iff $C(A|E, X_t^A) = C(A|X_t^A)$.

The language of this schema's instances is English, the schematic variables are *E* and *A* – ranging over designations of propositions –, C – ranging over designations of reasonable initial credence functions – and

X_t^A – ranging over designations of the proposition giving the expected (in C) chance$_t$ of *A*.

This gives us four candidate schemata: 1, 5, 8 and 9, with the latter two serving as exemplars for further schemata. For each of these there is an associated degree of relative admissibility definition schema in terms of resiliency over chances. Where X_t^A is a schematic variable – as in schemata 8 and 9 – it is easy to proceed by directly co-opting Skyrms' definition of resiliency:

8. *E* is admissible$_t$ for *A* to degree $\boldsymbol{\alpha}$ iff $\forall C[1 - |C(A|E, X_t^A) - C(A|X_t^A)| = \alpha]$.

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the schematic variables are *E* and *A* – ranging over designations of propositions –, α – ranging over designations of reals on the unit interval – and X_t^A – ranging over designations of the proposition giving the actual chance$_t$ of *A*.

9. *E* is admissible$_t$ for *A* in $\boldsymbol{C}$ to degree $\boldsymbol{\alpha}$ iff $1 - |C(A|E, X_t^A) - C(A|X_t^A)| = \alpha$.

The language of this schema's instances is English, the schematic variables are *E* and *A* – ranging over designations of propositions –, C – ranging over designations of reasonable initial credence functions, α – ranging over designations of reals on the unit interval – and X_t^A – ranging over designations of the proposition giving the expected (in C) chance$_t$ of *A*.

Where X_t^A is bound by a universal quantifier the task is more difficult, for the value of $C(A|E, X_t^A) - C(A|X_t^A)$ may vary depending upon the value that X_t^A takes. My proposal is to define degree of admissibility in terms of minimal resiliency given chances for such schemata: i.e., *E*'s degree of admissibility$_t$ for *A* is the least degree to which an X_t^A screens off *A* from *E*.

***1. E* is admissible$_t$ for *A* to degree $\boldsymbol{\alpha}$** iff

$$\forall C \begin{bmatrix} \exists X_t^A[1 - |C(A|E, X_t^A) - C(A|X_t^A)| = \alpha] \wedge \\ \forall X_t^A[\,1 - |C(A|E, X_t^A) - C(A|X_t^A)| \geq \alpha] \end{bmatrix}.$$

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the intended domain of X_t^A is the chance$_t$ of *A* propositions – $\{cht(A) = x : x \in [0,1]\}$ –, and the schematic variables are *E* and *A* – ranging over designations of propositions –, t – ranging over designations of times – and α – ranging over designations of reals on the unit interval.

5. *E* is admissible$_t$ for *A* in $\boldsymbol{C}$ to degree $\boldsymbol{\alpha}$ iff

$$\exists X_t^A[1 - |C(A|E, X_t^A) - C(A|X_t^A)| = \alpha] \wedge$$
$$\forall X_t^A[1 - |C(A|E, X_t^A) - C(A|X_t^A)| \geq \alpha].$$

The language of this schema's instances is English, the intended domain of X_t^A is the chance$_t$ of *A* propositions – $\{ch_t(A) = x : x \in [0,1]\}$ –, and the schematic variables are *E* and *A* – ranging over designations of propositions –, t – ranging over designations of times –, C – ranging over designations of reasonable initial credence functions – and α – ranging over designations of reals on the unit interval.

It can be easily checked for each of these schemata that they satisfy the criterion that *E* is admissible$_t$ for *A* iff *E* is admissible$_t$ for *A* to degree 1. The schemata 8 and 9 for degree of relative admissibility have the peculiar property that how admissible a soothsayer's proclamation about the future is depends upon the chance picked out by the condition in the side note. For instance, 8 implies that a soothsayer's prophecy that a coin to be flipped will land heads is less admissible than that same soothsayer's prophecy that a dice to be rolled will come up 6. The prophecies entail the outcomes so $C(H|P_H, X_t^H) = C(six|P_6, X_t^6) = 1$, but $C(H|P_H, X_t^H) = \frac{1}{2}$ and $C(six|P_6, X_t^6) = \frac{1}{6}$, in all initially reasonable C; hence the soothsayer's prophecy that the result of the coin toss will be heads is admissible to degree 1/2, while their prophecy that the dice roll will result in a six is admissible to degree 1/6. This is a decidedly peculiar way for degree's of admissibility to behave and reflects badly on not only the definitions of degree of relative admissibility canvassed above, but also their associated definitions of relative admissibility 8 and 9. Indeed, were the coin double-headed $(ch_t(H) = 1)$, the soothsayer's prophecy would be admissible$_t$ according to 8 precisely because of this chance relativity, and this is arguably also contrary to what one expects. Together with the fairly arbitrary nature of the side note conditions specifying to which chances the definitions are to be relative, one can argue that there is sufficient reason to reject definition schemata for relative admissibility and its degrees where X_t^A is a schematic variable. Accepting this argument – as I do – leaves only two candidate definition schemata conforming to the framework developed herein:

<u>The Objective Schema</u>:

E* is admissible$_t$ for *A iff $\forall C \forall X_t^A[C(A|E,X_t^A) = C(A|X_t^A)]$.

***E* is admissible$_t$ for *A* to degree $\boldsymbol{\alpha}$** iff

$$\forall C \begin{bmatrix} \exists X_t^A[1 - |C(A|E,X_t^A) - C(A|X_t^A)| = \alpha] \wedge \\ \forall X_t^A[1 - |C(A|E,X_t^A) - C(A|X_t^A)| \geq \alpha] \end{bmatrix}.$$

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the intended domain of X_t^A is the chance$_t$ of *A* propositions: $\{ch_t(A) = x : x \in [0,1]\}$. The schematic variables are *E* and *A* – ranging over designations of propositions –, *t* – ranging over designations of times – and α – ranging over designations of reals on the unit interval.

<u>The Subjective Schema</u>:

E* is admissible$_t$ for *A in C iff $\forall X_t^A[C(A|E,X_t^A) = C(A|X_t^A)]$.

E* is admissible$_t$ for *A* to degree $\boldsymbol{\alpha}$ *in C iff

$$\exists X_t^A[1 - |C(A|E,X_t^A) - C(A|X_t^A)| = \alpha] \wedge$$
$$\forall X_t^A[1 - |C(A|E,X_t^A) - C(A|X_t^A)| \geq \alpha]$$

The language of this schema's instances is English, the intended domain of C is the reasonable initial credence functions, the intended domain of X_t^A is the chance$_t$ of *A* propositions: $\{ch_t(A) = x : x \in [0,1]\}$. The schematic variables are *E* and *A* – ranging over designations of propositions –, *t* – ranging over designations of times – and α – ranging over designations of reals on the unit interval.

Understanding Admissibility and its Degrees

Both these schemata, the Objective and the Subjective, score reasonably well in terms of their continuity with the characterizations of relative admissibility to be found in the literature. They also improve on these aforementioned characterizations in their precision and clarity. Best of all, they both facilitate a natural definition of degree of relative admissibility; thereby, filling a lacuna in the literature. Finally, both schemata are perfectly coherent. Unfortunately, each has a flaw: the Subjective schema allows for highly peculiar extensions of the admissibility predicate, while the Objective schema is uninformative.

For any particular agent's reasonable initial credence function C, pair of propositions *A* and *E*, and time *t*, we can verify for a great many chances$_t$ of *A* that those chances screen off *A* from *E* in C. We can then make an induction to all such chances doing the same. This will give us a good, but defeasible, reason to believe *E* admissible$_t$ for *A* in C according to the definition and so we may ascertain which propositions are admissible$_t$ relative to each other for a particular subject. Alternatively, we can see whether *A* is independent of *E* in C and then make an *a fortiori* argument from such independence to independence conditional on the chances, and so to *E*'s admissibility$_t$ for *A* in C. Finally, it is a consequence of this definition schema that *E* is inadmissible$_t$ for *A* in any C that grants any credence to the entailment of *A* by *E*. In brief, we can use the definition to help sort the admissible$_t$ from the inadmissible$_t$ for any *A* in C, and this makes the definition informative. However, by the very nature of this definition what is admissible for one person may not be admissible for another. If a person is convinced that any coin they toss on Tuesday's is bound to land heads, then for that person, the proposition that it is Tuesday will be inadmissible for the proposition that the result of coin toss to be made is heads. Indeed, one can imagine any number of examples where an agent's peculiar, but rational, beliefs give rise to strange extensions of the admissibility predicate for them. So while the Subjective schema is informative, it is unacceptably subjective.

This leaves only the Objective definition schema for admissibility in terms of screening off by chances as viable; unfortunately, this schema is uninformative. Suppose we claim that *E* is admissible$_t$ for *A*, then according to the objective definition schema what we are claiming is that initial reasonable credence is such that $C(A|E, X_t^A) = C(A|X_t^A)$, for all X_t^A. But how are we to verify this? The above is not implied by other generally agreed principles of reasonable credence, is not supported by a dutch book argument and cannot be ascertained empirically. It seems that the only way to ascertain whether or not initial reasonable credence is like this is simply to stipulate that this is so. Consequently, this definition schema is, for the most part, uninformative. The qualification of the preceeding is there because it follows from the Objective definition schema that, if *E* entails *A*, then *E* is always inadmissible for *A*; hence knowledge of entailments implies knowledge of inadmissibilities by the objective definition making that definition conditionally informative to a limited extent.

But does a definition have to be informative for it to be of philosophical use? As there are other examples of uninformative definitions enjoying prominent positions in philosophy, the answer appears to be "No.". An example of such is the Platonic definition of knowledge as true, justified, belief. Famously, one cannot use this definition to identify what is known about the external world, for truth transcends any evidence one can have about the external world. I.e., there is no evidence for *P*, where *P* is about the external world, possessible by *X* such that *P* cannot be false. Consequently, Plato's definition of knowledge–which is often presented as a schema–is largely uninformative. Despite this shortcoming, philosopher's have found Plato's definition of knowledge to be illuminating even when applied to knowledge of the external world.

So it seems that whilst informativity is a virtue definitions should aspire to, uninformative definitions still have their uses in philosophy; particularly in the clarification of meaning. It is in this spirit that I offer the Objective definition schema for relative admissibility and its degrees. While it is admitted that this schema is largely useless at settling disputes over the extension of the admissibility predicate, it is hoped that the increase in our understanding of Lewis' admissibility gleaned from this definition schema is sufficient justification for its endorsement.

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